2007 IEEE International Conference on Control and Automation
Guangzhou, CHINA - May 30 to June 1, 2007

ThC4-6

Robust Moving Horizon Estimation for Constrained Linear System with Uncertainties

Haiyan Zhao, H.Chen,
Shuyou Yu and Guangxin Han
Department of Control Science
and Engineering, Jilin University,
Campus Nanling, 130025 Changchun, PR China
e-mail: haiyan@email.jlu.edu.cn

Guangxin Han
Department of Automation,
Jilin Institute of Chemical Technology,
Jilin 132022, China

***Abstract*—This paper presents a tractable robust moving horizon estimation (MHE) scheme, where the to be on-line solved optimization problem is relaxed to a minimization problem with an guaranteed bound for any allowed uncertainty. This proposed approach can make use of the additional knowledge of constraints on states and disturbances to achieve an improvement in the estimation performance. Simulation results show that the robust MHE is effective for constrained linear systems with uncertain model.**

Keywords-time domain constraint;moving horizon estimation; uncertain system

I. INTRODUCTION

In the past three decades, estimation problem has attracted the interests of many researchers and one of the popular methods is based on the minimization of the variance of the estimation error, i.e. the celebrated Kalman filtering approach [2]. But a central premise in the Kalman filtering theory is that state-space model is accurate and no constraints on states and disturbances. As these assumptions are not easily satisfied in practice, the standard Kalman filter may not be robust against model uncertainty and disturbances or the performance of the filter can deteriorate appreciably [7]. So research efforts were focused on approaches that do not rely on such requirements. For example, an H_∞ filter is designed by imposing that the H_∞ norm of the mapping between the disturbances and the estimation error is minimum. A further possibility consists in minimizing a quadratic cost function that penalizes the differences between the measures and the corresponding predictions, thus leading to the so-called least-squares estimation. As to the robustness with respect to system uncertainties for H_∞ estimators, the reader is referred to [4]. In addition, many robust filtering algorithms, such as min-max recursive robust filter, set-valued estimation, filtering and guaranteed cost paradigm, has attracted much attention, see e.g., ([7,12,13]). In practice, often additional insight about the process is available in the form of inequality constraints, such as the concentration of liquid is plus. Here the goal is that of developing a method that provides robust minimum-variance state estimates for uncertain constrained linear discrete-time systems according to a moving horizon approach.

Building on the success of moving horizon control, moving horizon estimation (MHE) has been suggested as a practical strategy to incorporate inequality constraints in estimation, e.g., ([10,6,3]). The basic strategy of the moving horizon approximation is to consider explicitly a fixed amount of data, while approximately summarizing the old data not explicitly accounted for by the estimator. Rao and Rawlings proved stability for moving horizon estimation of the constrained linear system and have demonstrated that MHE is a practical strategy for constrained state estimation [6].

Despite the vast literature on moving horizon state estimation, few results on the robustness of such methods is known to the authors. This motivates our efforts in addressing robustness to system uncertainty for the moving horizon estimation [1]. In this paper, we propose robust MHE strategy for the constrained system with norm bounded parameter uncertainty in both the state and output matrices. Robust MHE is, in general, formulated as solving a constrained minimax (instead of the minimization) problem on-line, where the maximization is performed over a set of uncertainties and/or disturbances. But the tractability is a crucial issue of minimax MHE schemes. Here, we firstly find a guaranteed upper bound for any allow uncertainties. Then, we make use of the additional knowledge of constraints on states and disturbances to achieve an improvement in the estimation performance by searching a scalar factor.

This paper is organized as follows. Section II proposes the problem to be studied. The development and formulation of the proposed robust MHE are presented in Section III. A numerical example is illustrated in Section IV, which shows the feasibility of this approach.

II. PROBLEM STATEMENT

Consider constrained discrete-time system with uncertainties as follows:

$$\begin{aligned} x_{k+1} &= (A+\Delta A_k)x_k + Bw_k \\ y_k &= (C+\Delta C_k)x_k + v_k \end{aligned} \tag{1}$$

subject to the following time-domain constraints:

$$x_k \in \mathbb{X}, w_k \in \mathbb{W}, \quad \upsilon_k \in \mathbb{V} \tag{2}$$

$x_k \in \Re^n$ is the system state, $y_k \in \Re^m$ is the measurement, $w_k \in \Re^p$ and $v_k \in \Re^m$ are system and measurement noise sequences respectively that satisfy $\mathbb{W}$, $\mathbb{X}$ and $\mathbb{V}$ are polyhedral and convex, the process and measurement noises has the following assumption:

$$\begin{aligned} &E\{w(k)\}=0,\ E\{v(k)\}=0 \\ &E\{w(k)v^T(k)\}=0,\ E\{w(j)^T w(k)\}=W\delta_{jk} \\ &E\{v(j)v^T(k)\}=R\delta_{jk},\ j,k=0,1,2\cdots \end{aligned} \tag{3}$$

1-4244-0818-0/07/$20.00 © 2007 IEEE

where $E(\cdot)$ denotes the expectation and $\delta(k)$ is the Kronecker Delta function. A,B and C are known real matrices with appropriate dimensions, and ΔA_k , ΔC_k are unknown matrices which represent time-varying parameter uncertainties. These uncertainties are assumed to be of the following structure:

$$\begin{bmatrix} \Delta A_k \\ \Delta C_k \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} F_k E, \qquad F_k^T F_k \le I(k \ge 0) \tag{4}$$

where $F_k \in \Re^{i\times j}$ is an unknown real time-varying matrix, and H_1, H_2 and E are known real constant matrices of appropriate dimensions that specify how the elements of A and C are affected by uncertainty in F_k .

We assume that the system (1) is quadratically stable. To be more precise, the following definition is introduced.

Definition 1: The system (1) is said to be quadratically stable if there exists a symmetric positive definite matrix P such that

$$[A+\Delta A_k]^T P[A+\Delta A_k] - P < 0, \quad k = 0,1,2,\cdots$$

for all admissible uncertainties ΔA_k .

Moreover, we assume our estimation to be based on data obtained in the recent past according to a moving horizon strategy. We shall follow the moving horizon strategy described in [12] for quite a general setting and specialized in [6] for constrained linear systems with no uncertainties. More specifically, at any stage T the objective is to find estimates of the state vector $\hat{x}_{T-N}$ (N is the moving horizon size) on the basis of the observations vector $Y_{T-N}^N := \{ y_{T-N}, y_{T-N+1}, \cdots y_T \}$ and of the prior estimate state $\hat{x}_{T-N}$. Toward this end, we introduce the following cost function.

$$\begin{aligned} J_t = \min_{\hat{x}_{T-N}, W_{T-N}^{T-1}} & \sum_{k=T-N}^{T-1} v_k^T R^{-1} v_k + w_k^T Q^{-1} w_k \\ & + \left(\hat{x}_{T-N} - \bar{x}_{T-N}\right)^T S_{T-N}^{-1} \left(\hat{x}_{T-N} - \bar{x}_{T-N}\right) \end{aligned} \tag{5}$$

subject to (1) and (2), $v_k = y_k - (C+\Delta C_k)x_k$, $W_{T-N}^{T-1} := \{ w_{T-N}, w_{T-N+1} \cdots, w_{T-1} \}$. N is the moving horizon size. $\hat{x}_{T-N}$ and W_{T-N}^{T-1} are the decisional variables of the optimization. Q,R are symmetric positive matrix, which indicate the confidence of model disturbance and measurements of noises. S_{T-N} is error covariance matrix, which expresses our belief in the prior estimation state $\bar{x}_{T-N}$ as compared with the observation model.

The following section will introduce the algorithm of robust moving horizon estimation.

III. ROBUST MOVING HORIZON ESTIMATION

The basis of moving horizon estimation is the on-line solving of an optimization problem with constraints, updated by the actual measurements at each sampling time. In robust MHE, we strive in general to solve the following optimization problem for the system (1) with the actual measurement Y_{T-N}^T in the moving horizon fashion.

Problem 1:For a given pair $\left(\bar{x}_{T-N}, Y_{T-N}^T\right)$, find the optimal estimate:

$$\left(\hat{x}_{T-N}^*, \left\{\hat{w}_k^*\right\}_{k=T-N}^{T-1}\right) = \arg \min_{\hat{x}_{T-N}, W_{T-N}^{T-1}} J_t\left(\bar{x}_{T-N}, \Delta A_{T-N}^{T-1}, \Delta C_{T-N}^T\right) \tag{6}$$

Subject to (1) and (2), $v_k = y_k - (C+\Delta C_k)x_k$.

We can obtain the optimal solution $\left(\hat{x}_{T-N}^*, \left\{\hat{w}_k^*\right\}_{k=T-N}^{T-1}\right)$ at time T by solving Problem 1, then the state estimation can be obtained as follows:

$$\begin{aligned} \hat{x}_{T-N+1+i}^* &= (A+\Delta A_{T-N+i})\hat{x}_{T-N+i}^* + B\hat{w}_{T-N+i}^* \\ & (i = 0, \cdots N-1) \end{aligned} \tag{7}$$

Since the proposed estimation scheme is based on a minimization optimization with respect to the uncertainties in the system matrices, we solve the Problem 1 by

- Transform the uncertain mathematical problem into an equivalent certain mathematical program by appropriately searching a scaling design parameter;
- Estimate of ΔA_k is available by minimization of an upper bound on the worst-case cost;
- Find the optimal solution $\left(\hat{x}_{T-N}^*, \left\{\hat{w}_k^*\right\}_{k=T-N}^{T-1}\right)$ by minimizing the cost for a given pair $\left(\bar{x}_{T-N}, Y_{T-N}^T\right)$.

However, the main problem is how to obtain the prior estimate value $\bar{x}_{T-N}$ and the covariance matrix S_{T-N} .

A. *Estimation scheme*

First, we assume that the initial condition $\bar{x}_0$ is a zero mean Gaussian random variable independent of the noises w_k and v_k , and with an unknown covariance matrix that satisfies the following assumption.

Assumption 1:

- $E[\bar{x}_0 \bar{x}_0^T] \le \bar{S}_0$, where $\bar{S}_0 = \bar{S}_0^T > 0$ is a known matrix;
- $\text{rank}\begin{bmatrix} A & H_1 & BQ^{\frac{1}{2}} \end{bmatrix} = n$

Our first objective is to design a stable robust estimator of the form

$$\hat{x}_{k+1} = A_k \hat{x}_k + K_k y_k, \qquad \hat{x}_0 = 0 \tag{8}$$

where A_k and K_k are time-varying matrices to be determined in order that the variance of the estimation error ($e_k = x_k - \hat{x}_k$) is guaranteed to be smaller than a certain bound for all uncertainty matrices F_k satisfying (4) , i.e., the estimation error dynamics satisfies

$$E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] \le S_k$$

with S_k being an optimized upper bound of filtering covariance over the class of robust quadratic filters.

In terms of system (1), (4) and (8), the state-space estimation for the estimation error e_k are as follows:

$$\tilde{x}_{k+1} = (A_{cl} + H_{cl} F_k E_{cl})\tilde{x}_k + G\eta_k, \qquad \tilde{x}(0) = \tilde{x}_0 \tag{9}$$

$$e_k = L\tilde{x}_k \tag{10}$$

$$\tilde{x}_k = \begin{bmatrix} e_k \\ \hat{x}_k \end{bmatrix}, \quad \tilde{x}_0 = \begin{bmatrix} x_0 \\ 0 \end{bmatrix}, \quad \eta_k = \begin{bmatrix} w_k \\ v_k \end{bmatrix}, \quad L = \begin{bmatrix} I & 0 \end{bmatrix},$$

$$A_{cl}=\begin{bmatrix} A-K_kC & A-A_k-K_kC \\ K_kC & A_k+K_kC \end{bmatrix}, E_{cl}=\begin{bmatrix} E & E \end{bmatrix},$$
$$H_{cl}=\begin{bmatrix} H_1-K_kH_2 \\ K_kH_2 \end{bmatrix}, \quad G=\begin{bmatrix} B & -K_k \\ 0 & K_k \end{bmatrix}.$$

where η_k is a zero-mean white noise signal.

Definition 2: The estimator (8) for system (1) is said to be a stable quadratic estimator associated with a symmetric nonnegative definite matrix X satisfies the inequality

$$\left(A_{cl}+H_{cl}F_kE_{cl}\right)X\left(A_{cl}+H_{cl}F_kE_{cl}\right)^T-X+GG^T\le 0 \quad (11)$$

for all uncertainties ΔA_k and ΔC_k satisfying (4).

The definition of quadratic estimator is an extension of the standard Kalman filter. Indeed, in our main result of the paper, we will show that the quadratic estimator is a modified Kalman filter where the uncertainties of the system matrices are appropriately accounted for in the filter structure. It is a principal task to transform the uncertain mathematical problem into an equivalent certain mathematical program. To be more precise, the following definition is introduced.

Definition 3: Filter (8) is said to be a robust quadratic filter if for some $\varepsilon_k>0$, there exists a bounded $\Pi_k=\Pi_k^T\ge 0$ that satisfies the following Riccati difference equation (RDE):

$$\begin{aligned}\Pi_{k+1}&=A_{cl}\Pi_kA_{cl}^T+\varepsilon_k^{-1}H_{cl}H_{cl}^T+G\bar{W}G^T\\&\quad+A_{cl}\Pi_kE_{cl}^T(\varepsilon_k^{-1}I-E_{cl}\Pi_kE_{cl}^T)^{-1}E_{cl}\Pi_kA_{cl}^T\end{aligned} \quad (12)$$

and such that $I-\varepsilon_kE_{cl}\Pi_kE_{cl}^T>0$, where $\Pi_0=diag\{S_0,0\}$ and $\bar{Q}=diag\{Q,R\}$.

From [13], we can know that for all admissible uncertainties, the covariance matrix satisfies the bound $E[\tilde{x}_k\tilde{x}_k^T]\le\Pi_k \quad \forall k\in[0,T]$. Furthermore,

$$E[e_ke_k^T]\le L\Pi_kL^T=\Pi_{11,k}, \qquad \forall k\in[0,T] \quad (13)$$

where $\Pi_{11,k}\in\Re^{n\times n}$ is the (1,1) block of the matrix Π_k and e_k is the estimation error.

B. ΔA_k, ΔC_k calculation

Before calculation ΔA_k, ΔC_k, we introduce the following two lemmas. Lemma 1 is the matrix converse theorem. Lemma 2 provides a approximate transform condition from uncertain system to certain system.

Lemma 1[11]: For any matrices X and Y of appropriate dimensions and any constant $\alpha>0$

$$(X^{-1}-\alpha^{-1}Y^TY)^{-1}=X+XY^T(\alpha I-YXY^T)^{-1}YX \quad (14)$$

Lemma 2[12]: Given matrices Y,H,E of appropriate dimensions and with Y symmetric, then

$Y+HFE+E^TF^TH^T<0$ for all F satisfying $F^TF\le I$, if and only if there exists a scalar $\varepsilon>0$ such that $Y+\varepsilon HH^T+\varepsilon^{-1}E^TE<0$.

The following two RDEs need to be introduced which is related to the content of Lemma 3.

$$\begin{aligned}P_{k+1}&=AP_kA^T+AP_kE^T(\tfrac{1}{\varepsilon_k}-EP_kE^T)^{-1}EP_kA^T\\&\quad+\tfrac{1}{\varepsilon_k}H_1H_1^T+BWB^T\end{aligned} \quad (15)$$

$$\begin{aligned}S_{k+1}&=AQ_kA^T-(AQ_kC^T+\tfrac{1}{\varepsilon_k}H_1H_2^T)(R_{\varepsilon_k}+CQ_kC^T)^{-1}\\&\quad\times(AQ_kC^T+\tfrac{1}{\varepsilon_k}H_1H_2^T)^T+\tfrac{1}{\varepsilon_k}H_1H_1^T+BWB^T\end{aligned} \quad (16)$$

From theorem 2.1 of [5], we know that a quadratic estimator will provide a known guaranteed upper bound for filtering error covariance. The following Lemma, which shows that the existence of P_k and S_k is guaranteed by the existence of Π_k to (12).

Lemma 3: Under Assumption 1, for a given filer of (8) and for some scalar $\varepsilon_k>0$

- the RDE(12) has a bounded solution Π_k over $[0,T]$ and such that $I-\varepsilon_kE_{cl}\Pi_kE_{cl}^T>0$, then there exists a bounded solution $P_k=P_k^T>0$ to the RDE(15) over $[0,T]$ for the same $\varepsilon_k>0$, and such that $P_k^{-1}-\varepsilon_kE^TE>0$.
- the RDE (15) has a bounded solution P_k over $[0,T]$ and such that $P_k^{-1}-\varepsilon_kE^TE>0$, then there exists a bounded solution $S_k=S_k^T>0$ to the RDE (16) over $[0,T]$ for the same $\varepsilon_k>0$ and such that $S_k^{-1}-\varepsilon_kE^TE>0$. Furthermore, $P_k\ge S_k>0$ over $[0,T]$.

A proof of Lemma 3 can be found in [9].

Remark 3.1: In general, the optimal solution Π_k of (12) should be of the following partitioned form: $\Pi_k=\begin{pmatrix}\Pi_{11,k} & \Pi_{12,k}\\ \Pi_{21,k} & \Pi_{22,k}\end{pmatrix}.=\begin{pmatrix}\Pi_{11,k} & 0\\ 0 & P_k-\Pi_{11,k}\end{pmatrix}$, where all blocks are $n\times n$ matrices, which is argued similar to the continuous-time case as in [8].

Since the proposed estimation scheme is based on a minimization optimization with respect to the uncertainties in the system matrices. We should firstly obtain ΔA_k, ΔC_k at every sample time throughout the following Theorem.

Theorem 1: Consider that the uncertain system (1) and (4) satisfies Assumptions 1. Then there exists a robust quadratic filter for the system that minimizes the bound on the error variance in (13) if and only if exists $\beta_k>0$ and

$$\min_{P_{k+1},S_{k+1},\varepsilon_k}\ \text{trace}(S_{k+1}) \text{ subject to (18),(19) and (20).} \quad (17)$$

$$\begin{bmatrix} AP_kA^T+\beta_kH_1H_1^T+BQB^T-P_{k+1}-\lambda & AP_kE^T \\ \& & EP_kE^T-\beta_k \end{bmatrix}<0 \quad (18)$$

$$\begin{bmatrix} \beta_kH_1H_1^T+BWB^T-S_{k+1}+AS_kA^T & \beta_kH_1H_2^T+AS_kC^T & AS_kE^T \\ \& & V+\beta_kH_2H_2^T & CS_kE^T \\ \& & \& & ES_kE^T-\beta_kI \end{bmatrix}<0 \quad (19)$$

$\&$ denotes transpose of matrix, $\beta_k=\frac{1}{\varepsilon_k}$.

$$0<S_k\le P_k \quad (20)$$

exists a solution $P_k=P_k^T>0$ and $S_k=S_k^T>0$ over $[0,T]$ with $P_0=\bar{S}_0$, such that $P_k^{-1}-\varepsilon_kE^TE>0$ and $S_k^{-1}-\varepsilon_kE^TE>0$. λ is a assistant factor. Moreover, the optimal guaranteed cost can be obtained by $E[(x_k-\hat{x}_k)(x_k-\hat{x}_k)^T]\le S_k$. Under this condition, an optimal quadratic guaranteed cost a priori filter is given by

$$\hat{x}_{k+1} = (A+\Delta A_k)\hat{x}_k + (AQ_kC^T + \varepsilon_k^{-1}H_1H_2^T)$$
$$(R_{\varepsilon k} + CQ_kC^T)^{-1}(y_k - (C+\Delta C_k)\hat{x}_k),\ \hat{x}_0 = 0 \quad (21)$$

where

$\Delta A_k = \varepsilon_k A S_k E^T (I - \varepsilon_k E S_k E^T)^{-1} E$,

$\Delta C_k = \varepsilon_k C S_k E^T (I - \varepsilon_k E S_k E^T)^{-1} E$.

Proof: Firstly, we suppose that there exists a robust quadratic filter for the uncertain system. It follows from Definition 3 that there exists a bounded solution $\prod_k \geq 0$ to (12). From Lemma 3, we can know that bounded positive definite solution (P_k, S_k) exists satisfying $P_k^{-1} - \varepsilon_k E^T E > 0$ *and* $S_k^{-1} - \varepsilon_k E^T E > 0$.

Let $\beta_k = \varepsilon_k^{-1}$, the RDEs (15)-(16) can be transformed as follow:

$$\begin{bmatrix} AP_kA^T + \beta_k H_1 H_1^T + BWB^T - P_{k+1} - \lambda & AP_kE^T \\ \& & EP_kE^T - \beta_k \end{bmatrix} < 0 \quad (22)$$

$$\begin{bmatrix} AQ_kA^T + \beta_k H_1 H_1^T + BWB^T - S_{k+1} & AQ_kC^T + \beta_k H_1 H_2^T \\ \& & V + \beta_k H_2 H_2^T + CQ_kC^T \end{bmatrix} < 0 \quad (23)$$

Then it follows from (23) that

$$\begin{bmatrix} \beta_k H_1 H_1^T + BWB^T - S_{k+1} & \beta_k H_1 H_2^T \\ \& & V + \beta_k H_2 H_2^T \end{bmatrix} + \begin{bmatrix} AQ_kA^T & AQ_kC^T \\ \& & CQ_kC^T \end{bmatrix} < 0 \quad (24)$$

where

$\begin{bmatrix} AQ_kA^T & AQ_kC^T \\ \& & CQ_kC^T \end{bmatrix} = \begin{bmatrix} A \\ C \end{bmatrix} Q_k \begin{bmatrix} A^T & C^T \end{bmatrix}$, $Q_k^{-1} = S_k^{-1} - \varepsilon_k E^T E$,

$R_{\varepsilon_k} = V + \varepsilon_k^{-1} H_2 H_2^T$, then we can obtain:

$$\begin{bmatrix} \beta_k H_1 H_1^T + BWB^T - S_{k+1} & \beta_k H_1 H_2^T & A \\ \beta_k H_2 H_1^T & V + \beta_k H_2 H_2^T & C \\ A^T & C^T & -Q_k^{-1} \end{bmatrix} < 0 \quad (25)$$

$$\begin{bmatrix} \beta_k H_1 H_1^T + BWB^T - S_{k+1} & \beta_k H_1 H_2^T \\ \& & V + \beta_k H_2 H_2^T \end{bmatrix} - \begin{bmatrix} A \\ C \end{bmatrix} (\beta_k^{-1} E^T E - S_k^{-1})^{-1} \begin{bmatrix} A^T & C^T \end{bmatrix} < 0 \quad (26)$$

Applying Lemma 1 to (26), we can obtain

$$\begin{bmatrix} \beta_k H_1 H_1^T + BWB^T - S_{k+1} & \beta_k H_1 H_2^T \\ \& & V + \beta_k H_2 H_2^T \end{bmatrix} + \begin{bmatrix} A \\ C \end{bmatrix} (S_k + S_k E^T (\beta_k I - E S_k E^T)^{-1} E S_k) \begin{bmatrix} A \\ C \end{bmatrix}^T < 0 \quad (27)$$

Apply Schur to (27), we can obtain (19).

Sufficiency. From Lemma 3, we can know that a bounded solution $0 < S_k \leq P_k$ to the LMI (17) exists. In view of definition 3 and Lemma 3, we can see that the filter (21) is a robust quadratic estimator with an upper bound of error covariance S_k .

Necessary. The proof about deriving the necessary condition on the filter for optimality of the upper bound on the above error variance is analogous to the proof in[13].

From Theorem 1, we can get $(\Delta A_k, \Delta C_k)$ at every sample time. The prior estimation state of Problem 1 can be obtained recursively according to (21). The error variance matrix can also be obtained approximately by solving LMI (17).

Remark 3.2: When solving the robust MHE problem 1, we can first solve the problem: $\min_{P_{k+1}, S_{k+1}, \varepsilon_k} \operatorname{trace}(S_{k+1})$ subject to LMI (18), (19) and (20). The difference between theorem 1 from theorem described in [13] is that we obtain the stability condition and solve the robust problem by using the LMI tools. We solve the problem of LMIs with uncertain factor ε_k and obtain the error variance matrix at every sample time. A approximate error covariance matrix (S_k) for the constrained system is obtained from (17).

Remark 3.3: We note that when the parameter uncertainty ε_k in system (21) disappears and estimation horizon $N = 1$ for the unconstrained system, the robust moving horizon estimator (7) reduces to the standard Kalman filter for the nominal system.

C. *Robust MHE algorithm*

From statement above, we now give the following moving horizon algorithm.

1) Initialization. Set $P_0 = S_0 = \overline{S}_0$, Q , R , $\overline{x}_0$ and horizon N .
2) For $T-1 \leq N$, solve the LMI optimization problem (17) to get $(\Delta A_k, \Delta C_k)$. For a given pair $(\overline{x}_{T-N}, Y_{T-N}^T)$, we can get the optimal solution $\left(\hat{x}_0^*, \{w_k^*\}_{k=0}^{T-1}\right)$ by solving Problem 1. At last, compute estimation value by (7).
3) For $T-1 > N$, solve the LMI optimization problem (17) to get $(\Delta A_k, \Delta C_k)$, approximate error covariance matrix S_{T-N} , and the priori estimation state $\overline{x}_{T-N}$ can be computed by (21). At last, we obtain the optimal solution $\left(\hat{x}_0^*, \{w_k^*\}_{k=0}^{T-1}\right)$ according to a given pair $(\overline{x}_{T-N}, Y_{T-N}^T)$.
4) At sample time T , compute the estimation value $\hat{x}_T^*$ according to equ. (7).
5) Prepare for the next computation: The next time prior estimation value can be obtained by (21), and the error covariance matrix S_{T-N+1} can be obtained approximately based on Equ. (17). Let $T-1 \leftarrow T$, adopt the new measurement y_T . Go back to Step 2.

IV. SIMULATION

We consider the following uncertain discrete-time system

$$x_{k+1} = \begin{bmatrix} 0 & -0.5 \\ 1 & \delta \end{bmatrix} x_k + \begin{bmatrix} -6 \\ 1 \end{bmatrix} w_k \quad (28)$$

$$y_k = \begin{bmatrix} -100 & 10 \end{bmatrix} x_k + v_k \quad (29)$$

$$\overline{S}_0 = \begin{bmatrix} 69.2 & -79.0 \\ -79.0 & 234.1 \end{bmatrix},\ \overline{x}_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

- Disturbance constraints: $w_k \geq 0$;

- State constraints: $x_{1,k} \in [-0.4 \quad 0.4]$, $x_{2,k} \in [-0.4 \quad 0.4]$.

where δ is an uncertain parameter satisfying $|\delta| \le 0.3$. Note that the system above is of the form of system (1), (4) with $H_1 = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$, $H_2 = 0, E = [0 \quad 0.03], Q = 10$ and $V = 1$, and $N = 10$. $\Delta = 0, \Delta = \pm 0.3$ are considered in the simulation respectively. Fig. 1-Fig. 6 are the simulation results, and comparison with robust Kalman filter is given. From the simulation results, it is obvious that the estimation value obtained based on robust Kalman filter overstep the constraints. While the estimation state obtained based on strategy of robust MHE is within the constraints bound. The robust MHE method based on the optimal strategy can deal with the constraints considering all parameter uncertainties and guarantee an upper bound on the filtering error covariance. So the performance of robust MHE algorithm is better than the robust Kalman filter.

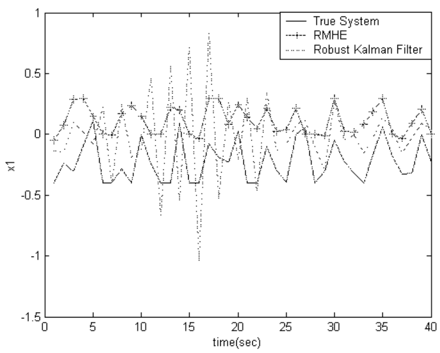

Fig. 1. Comparison of estimator x_1, ($\delta = -0.3$)

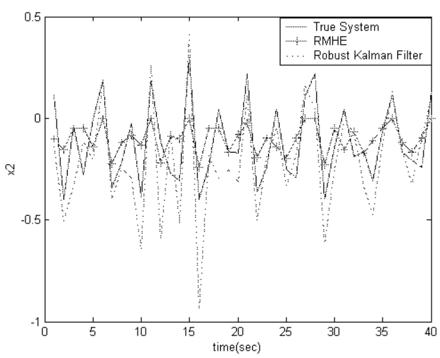

Fig. 2. Comparison of estimator x_2, ($\delta = -0.3$)

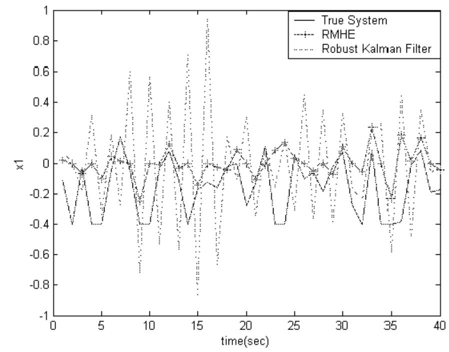

Fig. 3. Comparison of estimator x_1, ($\delta = 0$)

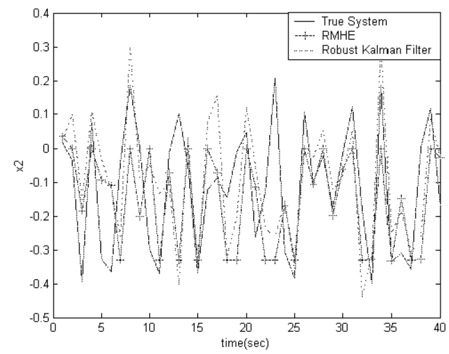

Fig. 4. Comparison of estimator x_2, ($\delta = 0$)

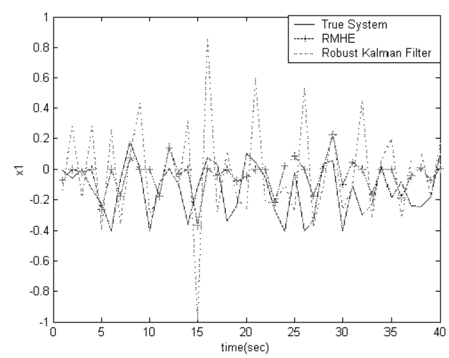

Fig. 5. Comparison of estimator x_1, ($\delta = 0.3$)

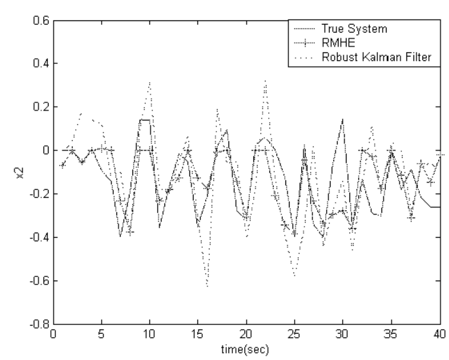

Fig. 6. Comparison of estimator x_2, ($\delta = 0.3$)

ACKNOWLEDGMENT

The authors would like to thank National Natural Science Foundation of P. R. China (60374027).

V. CONCLUSION

In the framework of moving horizon strategy, the robust estimation problem is formulated as a guaranteed cost problem subject to system dynamics and constraints on state and disturbance in this paper. Firstly, we obtain the prior estimation state based on a stable robust Kalman filter. Secondly, a approximate error covariance matrix is obtained based on LMIs. At the final, comparisons with robust Kalman filter are given. From the simulation results, we can know that this proposed approach can make use of the additional knowledge of constraints on states and disturbances to achieve an improvement in the estimation performance, so it is a practical and effective strategy for the constrained system with uncertain model.

REFERENCES

[1] G.Battistelli A. Alessandri, M. Baglietto. Robust receding –horizon state estimation for uncertain discrete-time linear systems. Automatica, 54:627-643, 2005.

[2] B. Anderson and J. Moore. Optimal filtering. N.J. Englewood Cliffis. 1979: Prentice-Hall.

[3] R. Findeisen and J. B. Rawlings. Suboptimal infinite horizon nonlinear model predictive control for discrete tiime systems. NATO Advanced sutdy institue on Nonlinear Model Based Process Control, 1997.

[4] Xie Lihua, David zhang. Improved robust H_2 / H_∞ filtering for uncertain discrete-time systems. Automatica, 40:873-880, 2004.

[5] Yeng chai soh, Lihua xie and E. de Souza. Robust kalman filtering for uncertain discrete-time systems. IEEE Trans. Automat. Contr,39:1310-1314, 1994.

[6] C. V. Rao, J. B. Rawlings and J. H. Lee. Constrained linear state estimation- a moving horizon approach. automatic, 2001. 37(10): p. 1619-1628.

[7] A. H. Sayed. A framework for stae-space estimation with uncertain models. IEEE Trans. Automat. Contr,46(7):998-1013, 2001.

[8] U. Shaked and C. E de Souza. Robust minimum variance filtering. IEEE Trans. Signal Process, 43:2474-2483, 1995.

[9] de Souza, C.E andShaked, U. and Fu, M. robust H∞filtering for continuous time varying uncertain systems with detrministic input signals. IEEE Trans on Signal Processing, 1995. **43**: p. 709-719.

[10] C. V. Rao, J. B. Rawlings and J. H. Lee. Nonlinear moving horizon state estimation. Proc. Int. Symp,, 1998: p. 146-163.

[11] Fuwen Yang Y. S. Hung. Robust H_∞ filtering with error variance constraints for discrete-time varying systems with uncertainty. Automatica,39:1185-1194, 2003.

[12] Xing Zhu; Yeng Chai Soh; Lihua Xie. Robust Kalman filter design. Decision and Control, 2000. Proceedings of the 39th IEEE Conference, 2000. 4: p. 3813 – 3818.

[13] Zhu, X.S., Yeng Chai; Xie, Lihua Design and analysis of discrete-time robust Kalman filters. Automatica 2002. **38**(6): p. 1069-1077.